Model comparison

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Topics

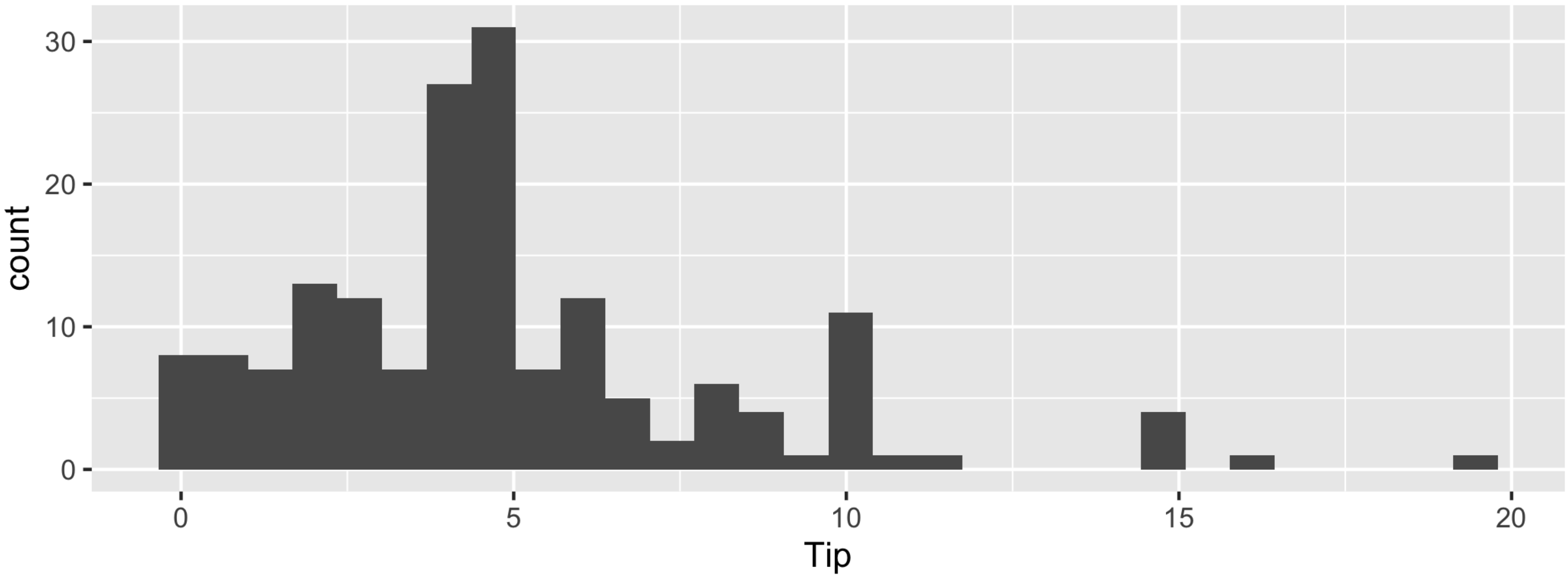
- ANOVA for Multiple Linear Regression
- Nested F Test
- R^2 vs. Adj. R^2
- AIC & BIC

Restaurant tips

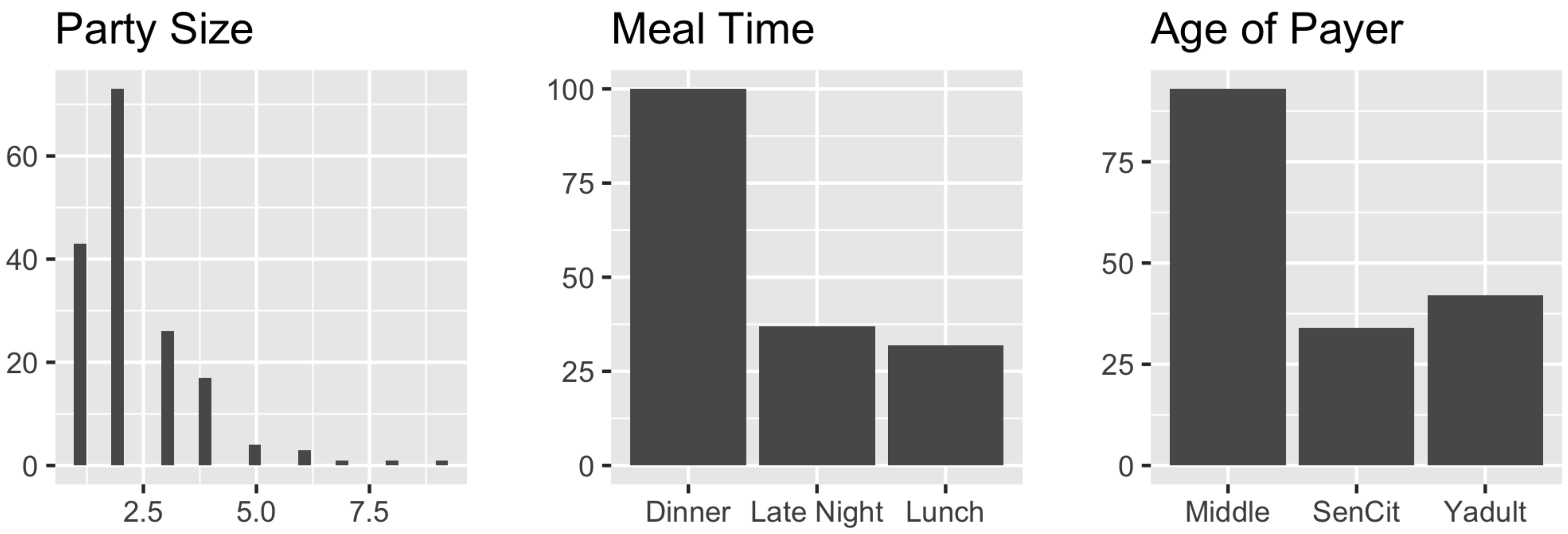
What affects the amount customers tip at a restaurant?

- **Response:**
  - `Tip`: amount of the tip
- **Predictors:**
  - `Party`: number of people in the party
  - `Meal`: time of day (Lunch, Dinner, Late Night)
  - `Age`: age category of person paying the bill (Yadult, Middle, SenCit)

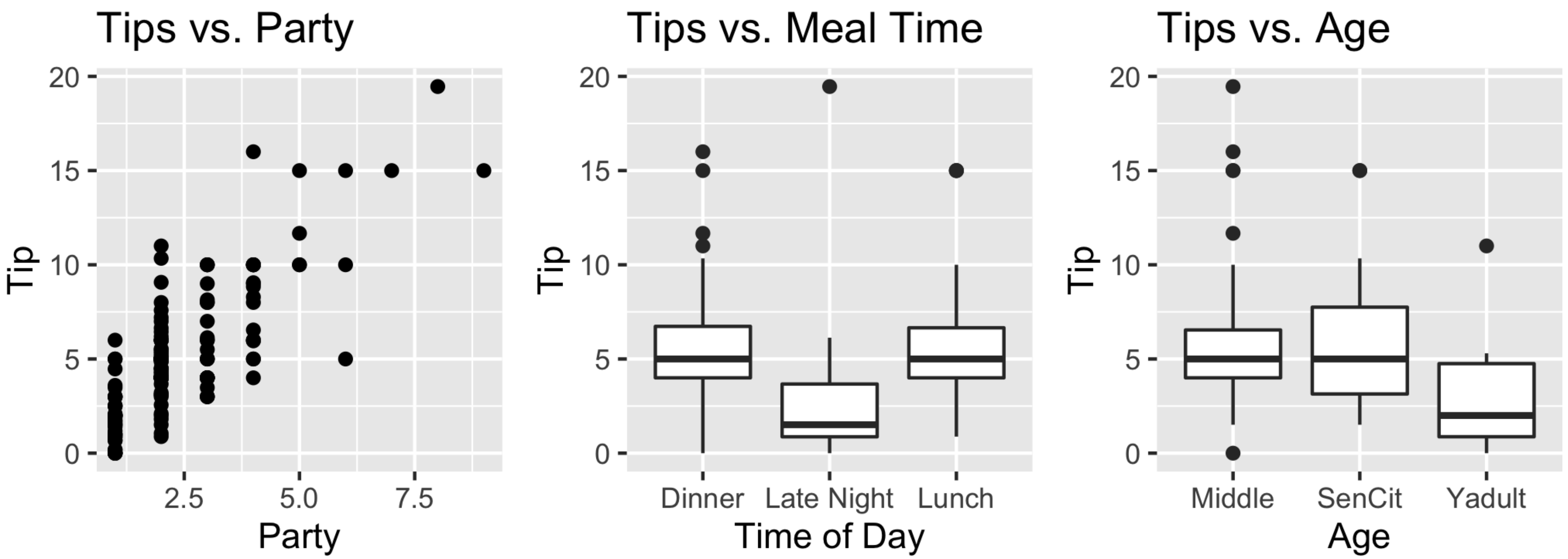
Response Variable



Predictor Variables



Response vs. Predictors



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Restaurant tips: model

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	0.838	0.397	2.112	0.036	0.055	1.622
Party	1.837	0.124	14.758	0.000	1.591	2.083
AgeSenCit	0.379	0.410	0.925	0.356	-0.430	1.189
AgeYadult	-1.009	0.408	-2.475	0.014	-1.813	-0.204

Is this the best model to explain variation in Tips?

ANOVA test for MLR

Using the ANOVA table, we can test whether any variable in the model is a significant predictor of the response. We conduct this test using the following hypotheses:

$$
\begin{aligned}
H_0 &: \beta_1 = \beta_2 = \cdots = \beta_p = 0 \\
H_a &: \text{at least one } \beta_j \text{ is not equal to 0}
\end{aligned}
$$

- The statistic for this test is the F test statistic in the ANOVA table
- We calculate the p-value using an F distribution with p and $(n - p - 1)$ degrees of freedom

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Tips: ANOVA Test

term	df	sumsq	meansq	statistic	p.value
Party	1	1188.636	1188.636	285.712	0.000
Age	2	38.028	19.014	4.570	0.012
Residuals	165	686.444	4.160		

Tips: ANOVA Test

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Model df: 3

Model SS: 1188.636 + 38.028 = 1226.664

Model MS: 1226.664/ 3 = 408.888

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Tips: ANOVA Test

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Model df: 3

Model SS: 1188.636 + 38.028 = 1226.664

Model MS: 1226.664/ 3 = 408.888

FStat: 408.888 / 4.160 = 98.2903846

P-value: P(F > 98.2903846) ≈ 0

Tips: ANOVA Test

term	df	sumsq	meansq	statistic	p.value
Party	1	1188.636	1188.636	285.712	0.000
Age	2	38.028	19.014	4.570	0.012
Residuals	165	686.444	4.160		

The data provide sufficient evidence to conclude that at least one coefficient is non-zero, i.e. at least one predictor in the model is significant.

Testing subset of coefficients

- Sometimes we want to test whether a **subset of coefficients** are all equal to 0

- This is often the case when we want test

  - whether a categorical variable with k levels is a significant predictor of the response
  - whether the interaction between a categorical and quantitative variable is significant

- To do so, we will use the **Nested (Partial) F Test**

Nested (Partial) F Test

- Suppose we have a full and reduced model:

$$\text{Full} : y = \beta_0 + \beta_1 x_1 + \cdots + \beta_q x_q + \beta_{q+1} x_{q+1} + \ldots \beta_p x_p$$
$$\text{Reduced} : y = \beta_0 + \beta_1 x_1 + \cdots + \beta_q x_q$$

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Nested (Partial) F Test

- Suppose we have a full and reduced model:

$$\text{Full} : y = \beta_0 + \beta_1 x_1 + \cdots + \beta_q x_q + \beta_{q+1} x_{q+1} + \ldots \beta_p x_p$$
$$\text{Reduced} : y = \beta_0 + \beta_1 x_1 + \cdots + \beta_q x_q$$

- We want to test whether any of the variables $x_{q+1}, x_{q+2}, \ldots, x_p$ are significant predictors. To do so, we will test the hypothesis:

$$H_0 : \beta_{q+1} = \beta_{q+2} = \cdots = \beta_p = 0$$
$$H_a : \text{at least one } \beta_j \text{ is not equal to } 0$$

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Nested F Test

- The test statistic for this test is

$$F = \frac{(SSE_{reduced} - SSE_{full})\Big/\text{\# predictors tested}}{SSE_{full}\Big/(n - p_{full} - 1)}$$

- Calculate the p-value using the F distribution with df1 = # predictors tested and df2 = $(n - p_{full} - 1)$

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Is `Meal` a significant predictor of tips?

term	estimate
(Intercept)	1.254
Party	1.808
AgeSenCit	0.390
AgeYadult	-0.505
MealLate Night	-1.632
MealLunch	-0.612

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Tips: Nested F test

$$H_0: \beta_{latenight} = \beta_{lunch} = 0$$
$$H_a: \text{ at least one } \beta_j \text{ is not equal to } 0$$

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Tips: Nested F test

$$H_0 : \beta_{latenight} = \beta_{lunch} = 0$$
$$H_a : \text{ at least one } \beta_j \text{ is not equal to } 0$$

```
reduced <- lm(Tip ~ Party + Age, data = tips)
```

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Tips: Nested F test

$$H_0 : \beta_{latenight} = \beta_{lunch} = 0$$
$$H_a : \text{ at least one } \beta_j \text{ is not equal to } 0$$

```
reduced <- lm(Tip ~ Party + Age, data = tips)
```

```
full <- lm(Tip ~ Party + Age + Meal, data = tips)
```

Tips: Nested F test

$$H_0 : \beta_{latenight} = \beta_{lunch} = 0$$
$$H_a : \text{ at least one } \beta_j \text{ is not equal to } 0$$

```
reduced <- lm(Tip ~ Party + Age, data = tips)
```

```
full <- lm(Tip ~ Party + Age + Meal, data = tips)
```

```
#Nested F test in R
anova(reduced, full)
```

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Tips: Nested F test

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
165	686.444				
163	622.979	2	63.465	8.303	0

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Tips: Nested F test

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
165	686.444				
163	622.979	2	63.465	8.303	0

F Stat: $\frac{(686.444-622.979)/2}{622.979/(169-5-1)} = 8.303$

Tips: Nested F test

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
165	686.444				
163	622.979	2	63.465	8.303	0

F Stat: $\frac{(686.444 - 622.979)/2}{622.979/(169-5-1)} = 8.303$

P-value: P(F > 8.303) = 0.0003

- calculated using an F distribution with 2 and 163 degrees of freedom

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Tips: Nested F test

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
165	686.444				
163	622.979	2	63.465	8.303	0

F Stat: $\frac{(686.444-622.979)/2}{622.979/(169-5-1)} = 8.303$

P-value: P(F > 8.303) = 0.0003

- calculated using an F distribution with 2 and 163 degrees of freedom

The data provide sufficient evidence to conclude that at least one coefficient associated with `Meal` is not zero. Therefore, `Meal` is a significant predictor of `Tips`.

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Model with `Meal`

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	1.254	0.394	3.182	0.002	0.476	2.032
Party	1.808	0.121	14.909	0.000	1.568	2.047
AgeSenCit	0.390	0.394	0.990	0.324	-0.388	1.168
AgeYadult	-0.505	0.412	-1.227	0.222	-1.319	0.308
MealLate Night	-1.632	0.407	-4.013	0.000	-2.435	-0.829
MealLunch	-0.612	0.402	-1.523	0.130	-1.405	0.181

Including interactions

Does the effect of **`Party`** differ based on the **`Meal`** time?

term	estimate
(Intercept)	1.276
Party	1.795
AgeSenCit	0.401
AgeYadult	-0.470
MealLate Night	-1.845
MealLunch	-0.461
Party:MealLate Night	0.111
Party:MealLunch	-0.050

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Nested F test for interactions

Let's use a Nested F test to determine if **`Party*Meal`** is statistically significant.

```
reduced <- lm(Tip ~ Party + Age + Meal, data = tips)
```

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```
reduced <- lm(Tip ~ Party + Age + Meal, data = tips)
```

```
full <- lm(Tip ~ Party + Age + Meal + Meal * Party,
           data = tips)
```

Nested F test for interactions

Let's use a Nested F test to determine if **`Party*Meal`** is statistically significant.

```
reduced <- lm(Tip ~ Party + Age + Meal, data = tips)
```

```
full <- lm(Tip ~ Party + Age + Meal + Meal * Party,
           data = tips)
```

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
163	622.979				
161	621.965	2	1.014	0.131	0.877

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Final model for now

We conclude that the effect of **`Party`** does not differ based **`Meal`**. Therefore, we will use the original model that only included main effects.

term	estimate	std.error	statistic	p.value
(Intercept)	1.254	0.394	3.182	0.002
Party	1.808	0.121	14.909	0.000
AgeSenCit	0.390	0.394	0.990	0.324
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Model comparision

R^2

Recall: R^2 is the proportion of the variation in the response variable explained by the regression model

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R^2 will always increase as we add more variables to the model

- If we add enough variables, we can always achieve $R^2 = 100\%$

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R^2

Recall: R^2 is the proportion of the variation in the response variable explained by the regression model

R^2 will always increase as we add more variables to the model

- If we add enough variables, we can always achieve $R^2 = 100\%$

If we only use R^2 to choose a best fit model, we will be prone to choose the model with the most predictor variables

Adjusted R^2

Adjusted R^2: measure that includes a penalty for unnecessary predictor variables

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Similar to R^2, it is a measure of the amount of variation in the response that is explained by the regression model

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Adjusted R^2

Adjusted R²: measure that includes a penalty for unnecessary predictor variables

Similar to R^2, it is a measure of the amount of variation in the response that is explained by the regression model

Differs from R^2 by using the mean squares rather than sums of squares and therefore adjusting for the number of predictor variables

R^2 and Adjusted R^2

$$R^2 = \frac{SS_{Model}}{SS_{Total}} = 1 - \frac{SS_{Error}}{SS_{Total}}$$

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R^2 and Adjusted R^2

$$R^2 = \frac{SS_{Model}}{SS_{Total}} = 1 - \frac{SS_{Error}}{SS_{Total}}$$

$$Adj.\,R^2 = 1 - \frac{SS_{Error}/(n-p-1)}{SS_{Total}/(n-1)}$$

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Using R^2 and $Adj. R^2$

$Adj. R^2$ can be used as a quick assessment to compare the fit of multiple models; however, it should not be the only assessment!

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Using R^2 and $Adj. R^2$

$Adj. R^2$ can be used as a quick assessment to compare the fit of multiple models; however, it should not be the only assessment!

Use R^2 when describing the relationship between the response and predictor variables

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Tips: Comparing models

Let's compare two models:

```
model1 <- lm(Tip ~ Party + Age + Meal, data = tips)
glance(model1) %>% select(r.squared, adj.r.squared)
```

```
## # A tibble: 1 x 2
##   r.squared adj.r.squared
##       <dbl>         <dbl>
## 1     0.674         0.664
```

```
model2 <- lm(Tip ~ Party + Age + Meal + Day, data = tips)
glance(model2) %>% select(r.squared, adj.r.squared)
```

```
## # A tibble: 1 x 2
##   r.squared adj.r.squared
##       <dbl>         <dbl>
## 1     0.683         0.662
```

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AIC & BIC

Akaike's Information Criterion (AIC):

$$AIC = n\log(SS_\text{Error}) - n\log(n) + 2(p+1)$$

Schwarz's Bayesian Information Criterion (BIC)

$$BIC = n\log(SS_\text{Error}) - n\log(n) + log(n) \times (p+1)$$

See the supplemental note on AIC & BIC for derivations.

AIC & BIC

$$
\begin{aligned}
AIC &= n\log(SS_\text{Error}) - n\log(n) + 2(p+1) \\
BIC &= n\log(SS_\text{Error}) - n\log(n) + \log(n)\times(p+1)
\end{aligned}
$$

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AIC & BIC

$$AIC = n\log(SS_{\text{Error}}) - n\log(n) + 2(p+1)$$
$$BIC = n\log(SS_{\text{Error}}) - n\log(n) + \log(n) \times (p+1)$$

First Term: Decreases as *p* increases

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AIC & BIC

$$AIC = n\log(SS_{\text{Error}}) - n\log(n) + 2(p+1)$$
$$BIC = n\log(SS_{\text{Error}}) - n\log(n) + \log(n) \times (p+1)$$

Second Term: Fixed for a given sample size *n*

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AIC & BIC

$$AIC = n\log(SS_{\text{Error}}) - n\log(n) + 2(p+1)$$
$$BIC = n\log(SS_{\text{Error}}) - n\log(n) + \log(n)\times(p+1)$$

Third Term: Increases as *p* increases

Using AIC & BIC

$$AIC = n\log(SS_{Error}) - n\log(n) + 2(p+1)$$
$$BIC = n\log(SS_{Error}) - n\log(n) + \log(n)\times(p+1)$$

- Choose model with the smaller value of AIC or BIC
- If $n \geq 8$, the penalty for BIC is larger than that of AIC, so BIC tends to favor *more parsimonious* models (i.e. models with fewer terms)

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Tips: AIC & BIC

```
model1 <- lm(Tip ~ Party + Age + Meal, data = tips)
glance(model1) %>% select(AIC, BIC)
```

```
## # A tibble: 1 x 2
##     AIC   BIC
##   <dbl> <dbl>
## 1  714.  736.
```

```
model2 <- lm(Tip ~ Party + Age + Meal + Day, data = tips)
glance(model2) %>% select(AIC, BIC)
```

```
## # A tibble: 1 x 2
##     AIC   BIC
##   <dbl> <dbl>
## 1  720.  757.
```

Recap

- ANOVA for Multiple Linear Regression
- Nested F Test
- R^2 vs. Adj. R^2
- AIC & BIC