

Simple Linear Regression

Conditions

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Topics

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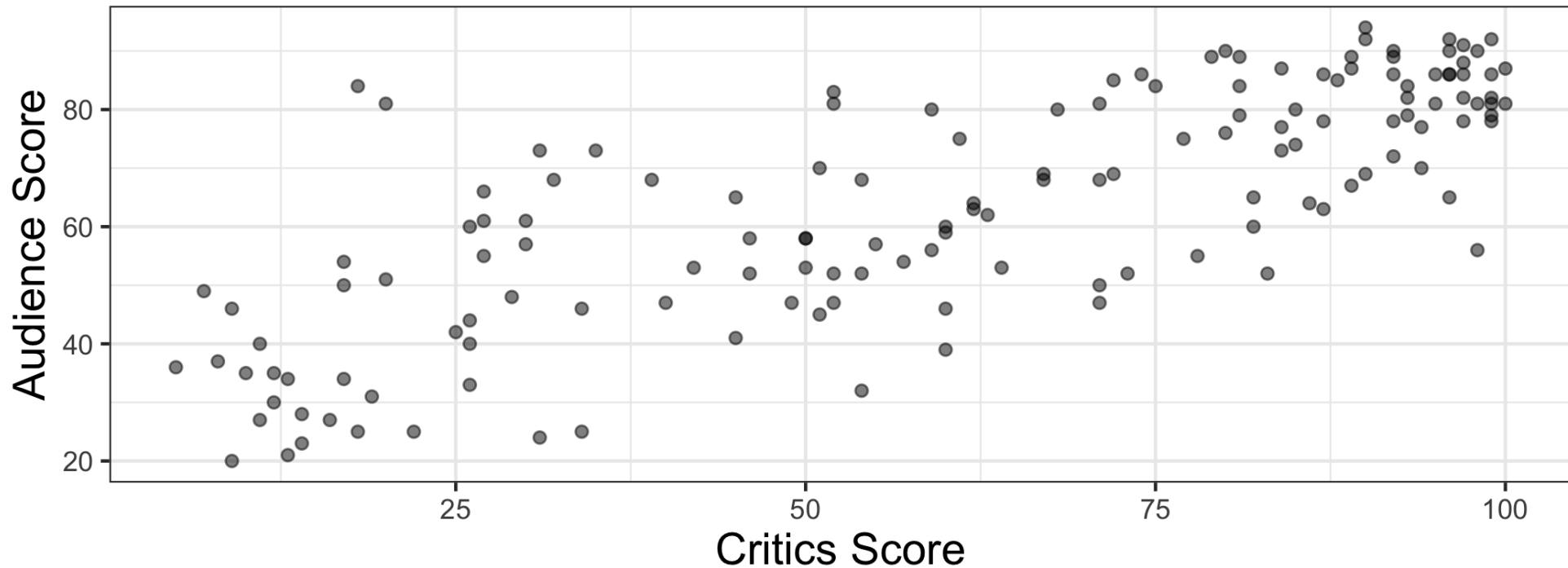
- List the conditions for simple linear regression

Topics

- List the conditions for simple linear regression
- Use plots of the residuals to check the conditions

Movie ratings data

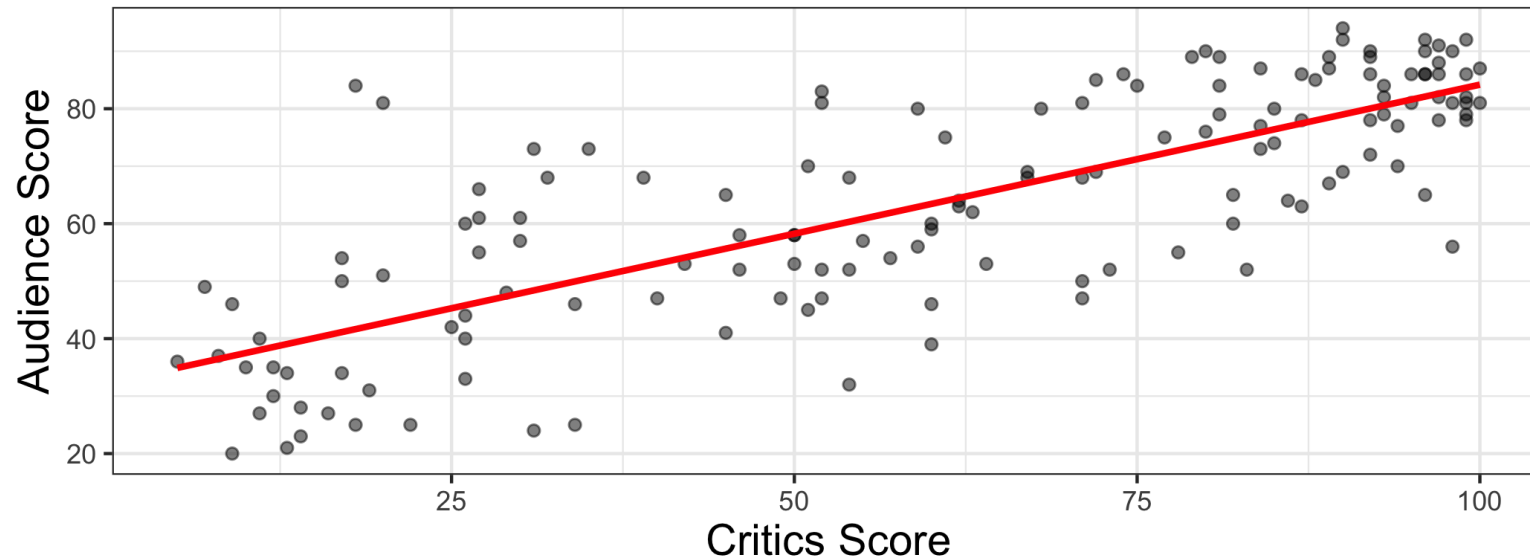
The data set contains the "Tomatometer" score (**critics**) and audience score (**audience**) for 146 movies rated on rottentomatoes.com.



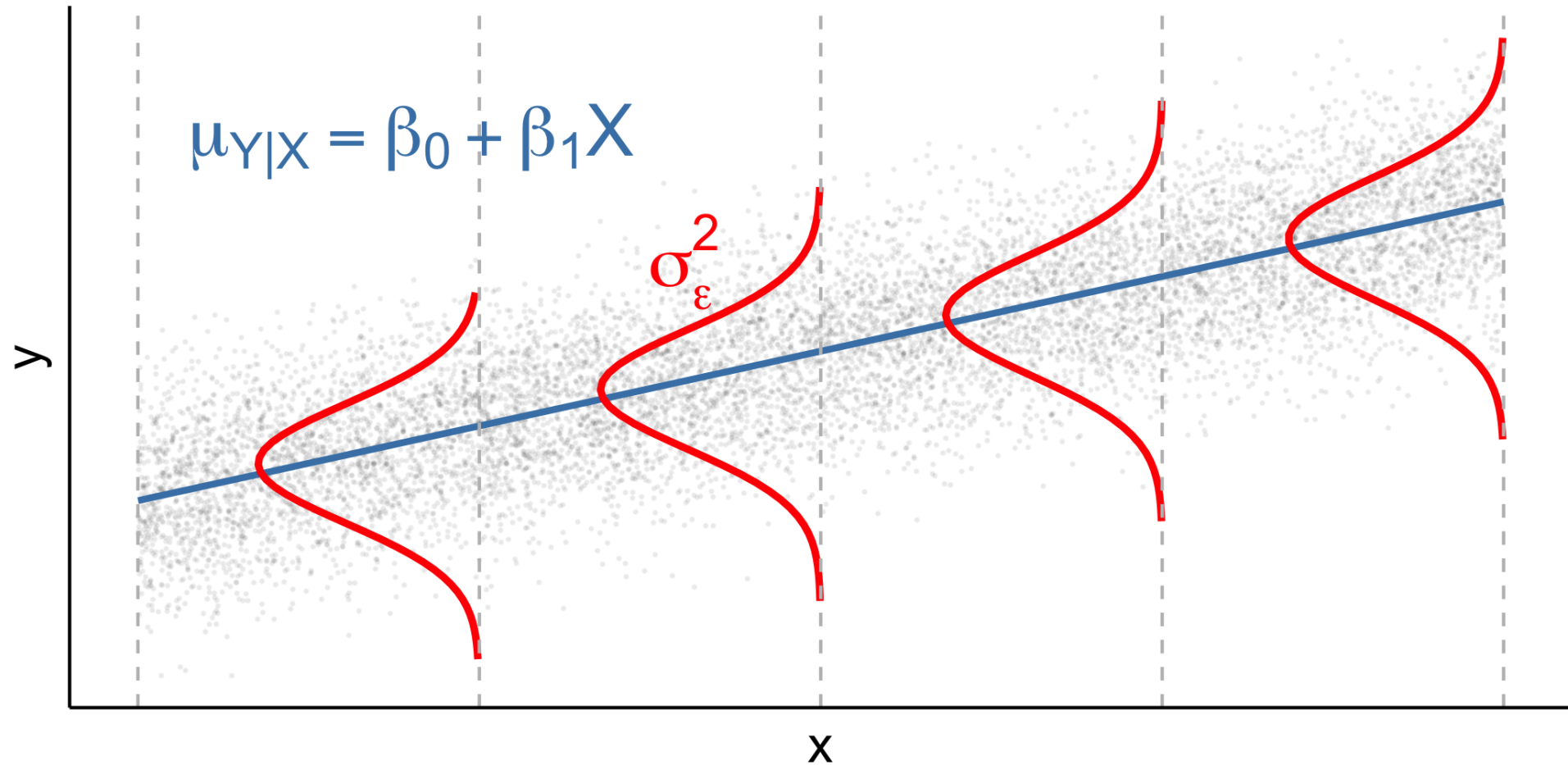
The model

$$\hat{\text{audience}} = 32.316 + 0.519 \times \text{critics}$$

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0



$$Y|X \sim N(\beta_0 + \beta_1 X, \sigma_\epsilon^2)$$



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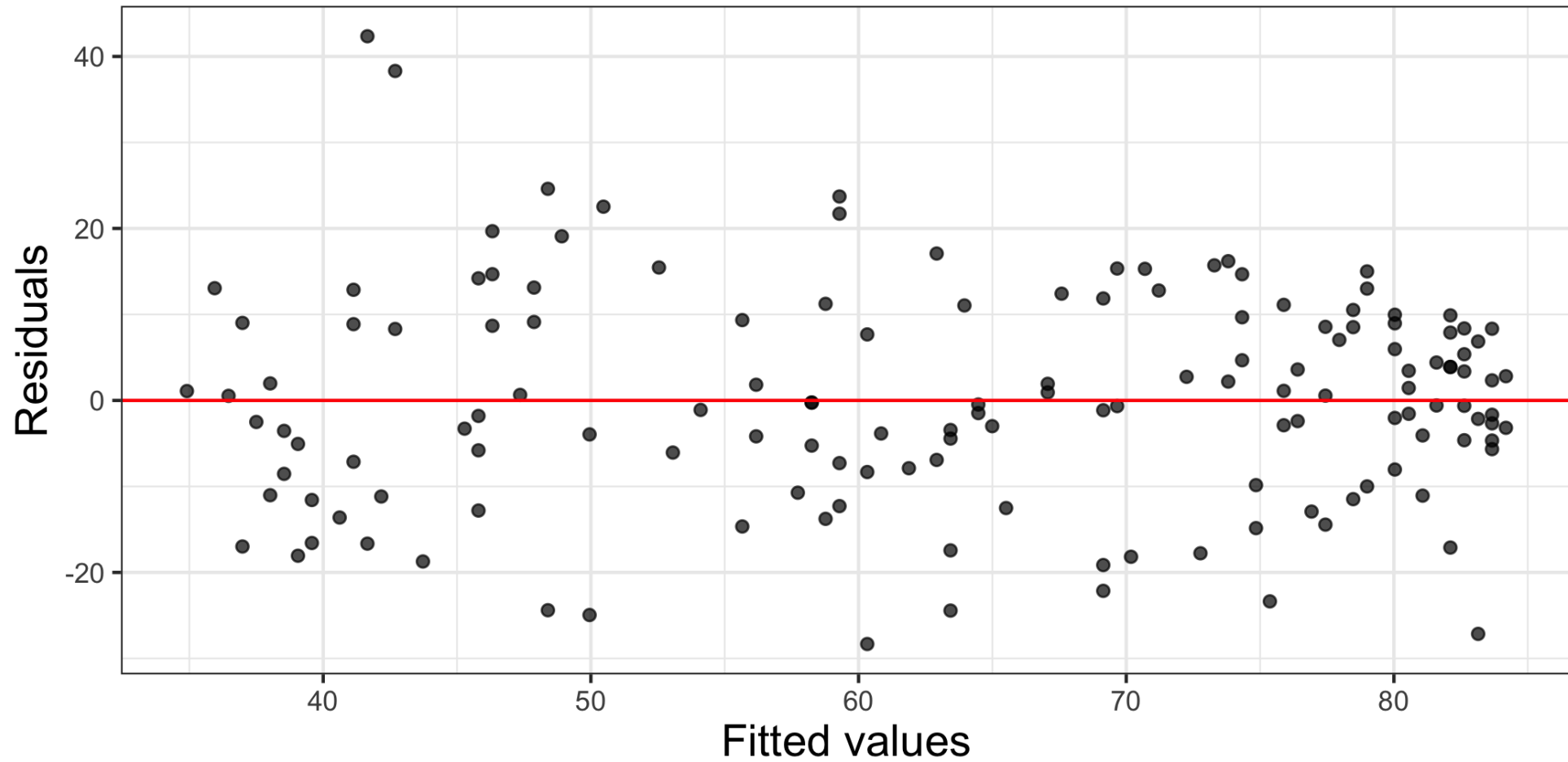
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1. **Linearity:** There is a linear relationship between the response and predictor variable.
2. **Constant Variance:** The variability of the errors is equal for all values of the predictor variable.
3. **Normality:** The errors follow a normal distribution.
4. **Independence:** The errors are independent from each other.

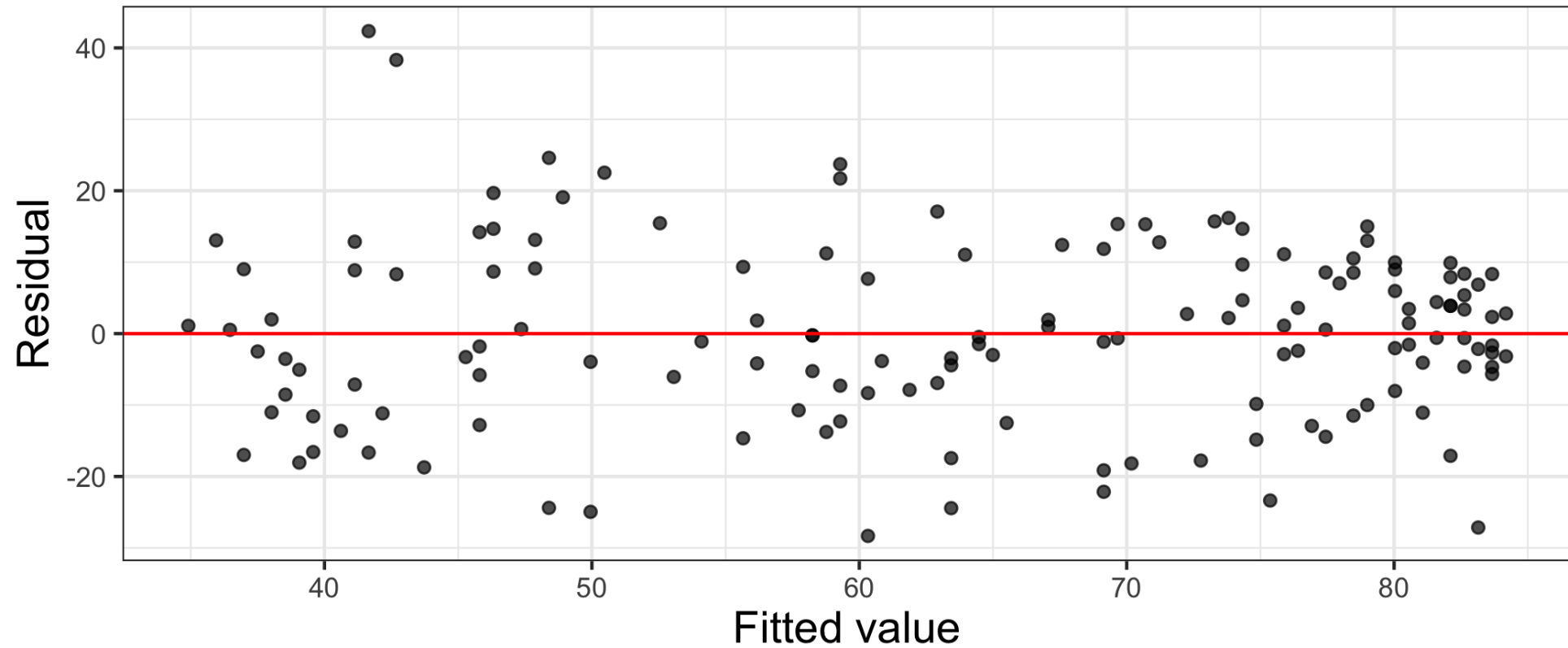
$$\text{residual}_i = e_i = y_i - \hat{y}_i$$

Residuals vs. fitted values

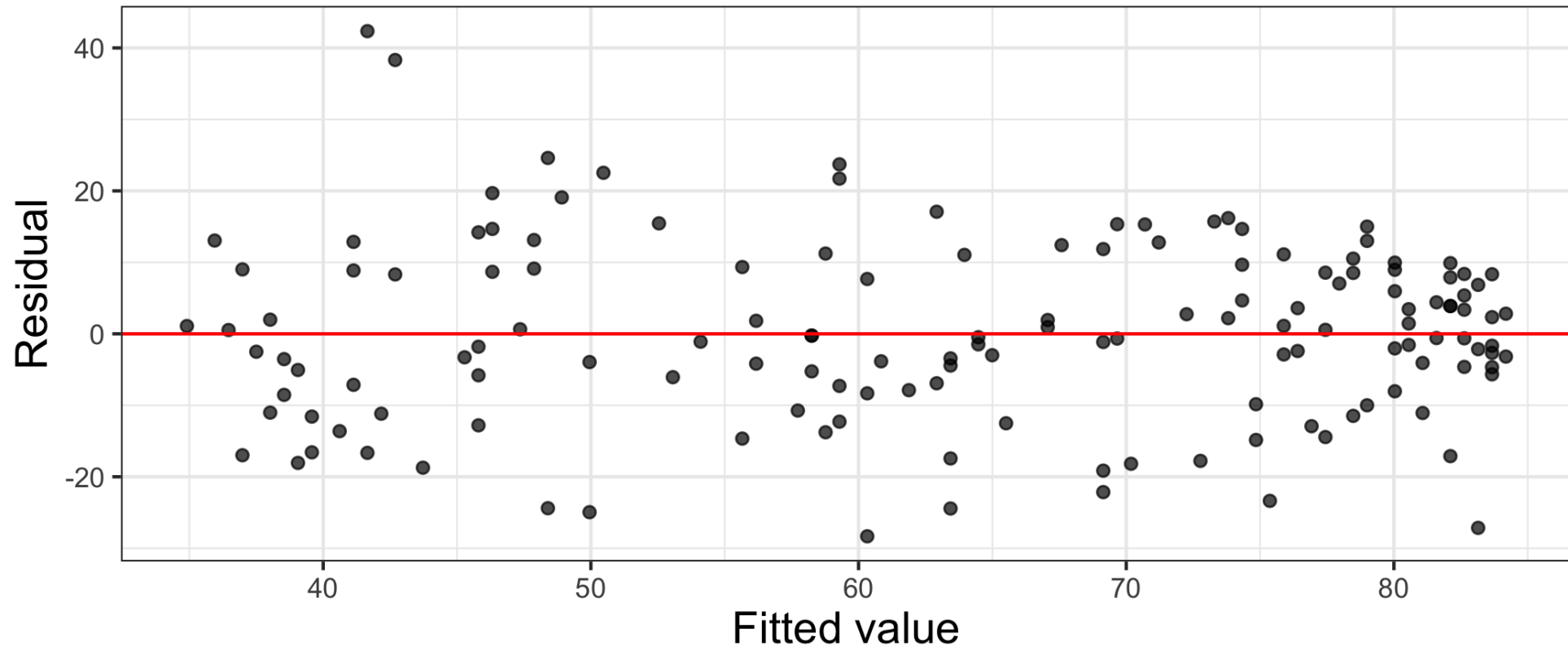


Checking linearity

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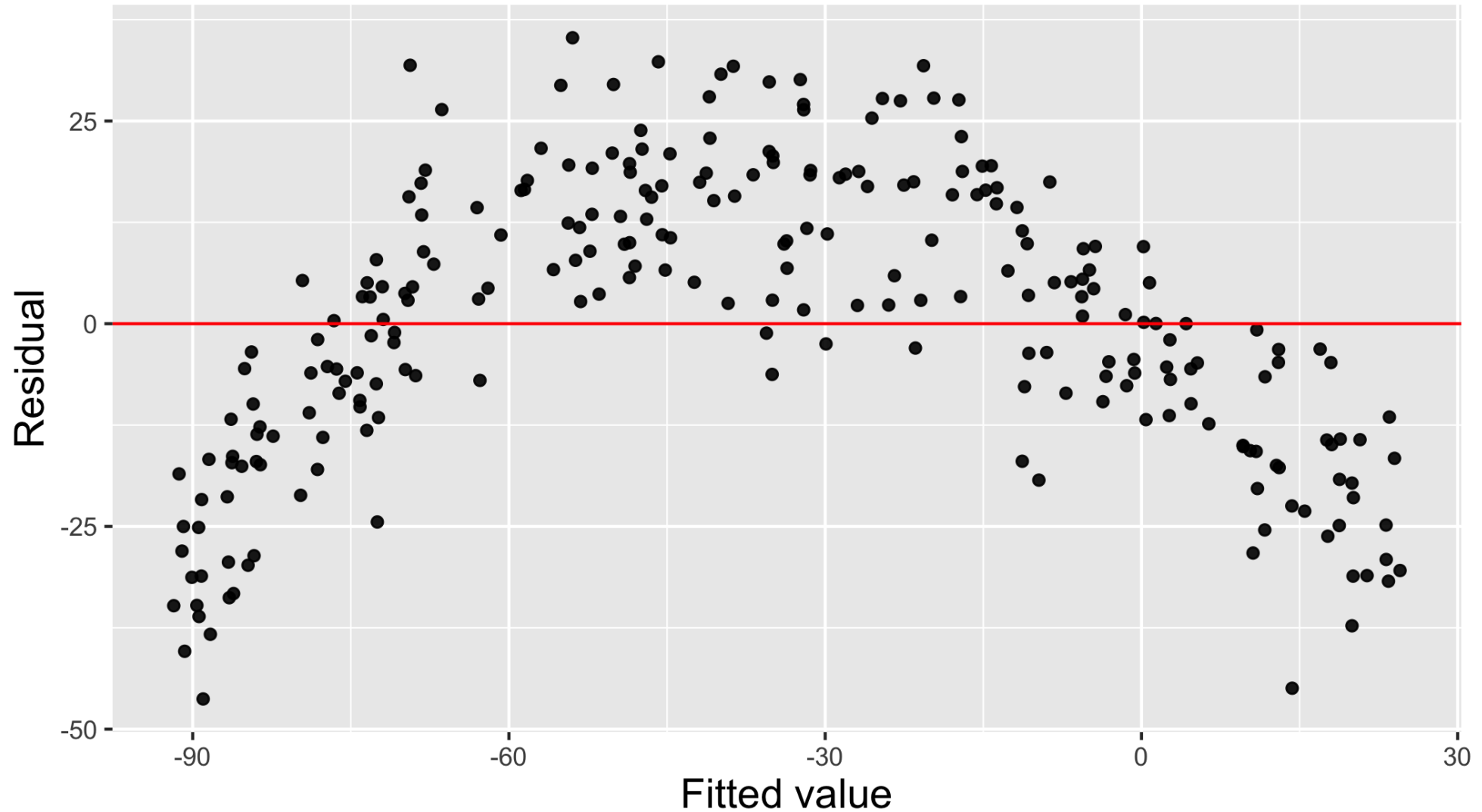


Checking linearity



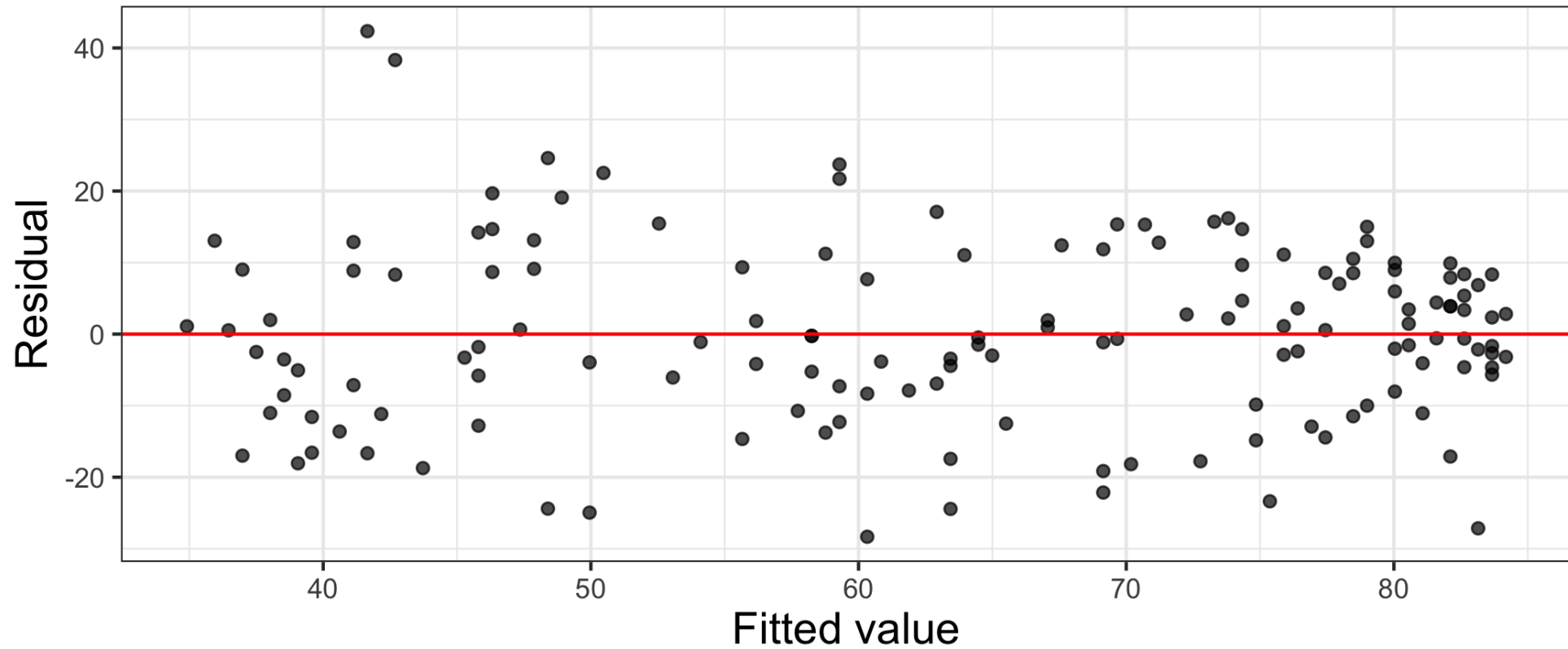
✅ There is no distinguishable pattern or structure. The residuals are randomly scattered.

✗ Violation: distinguishable pattern

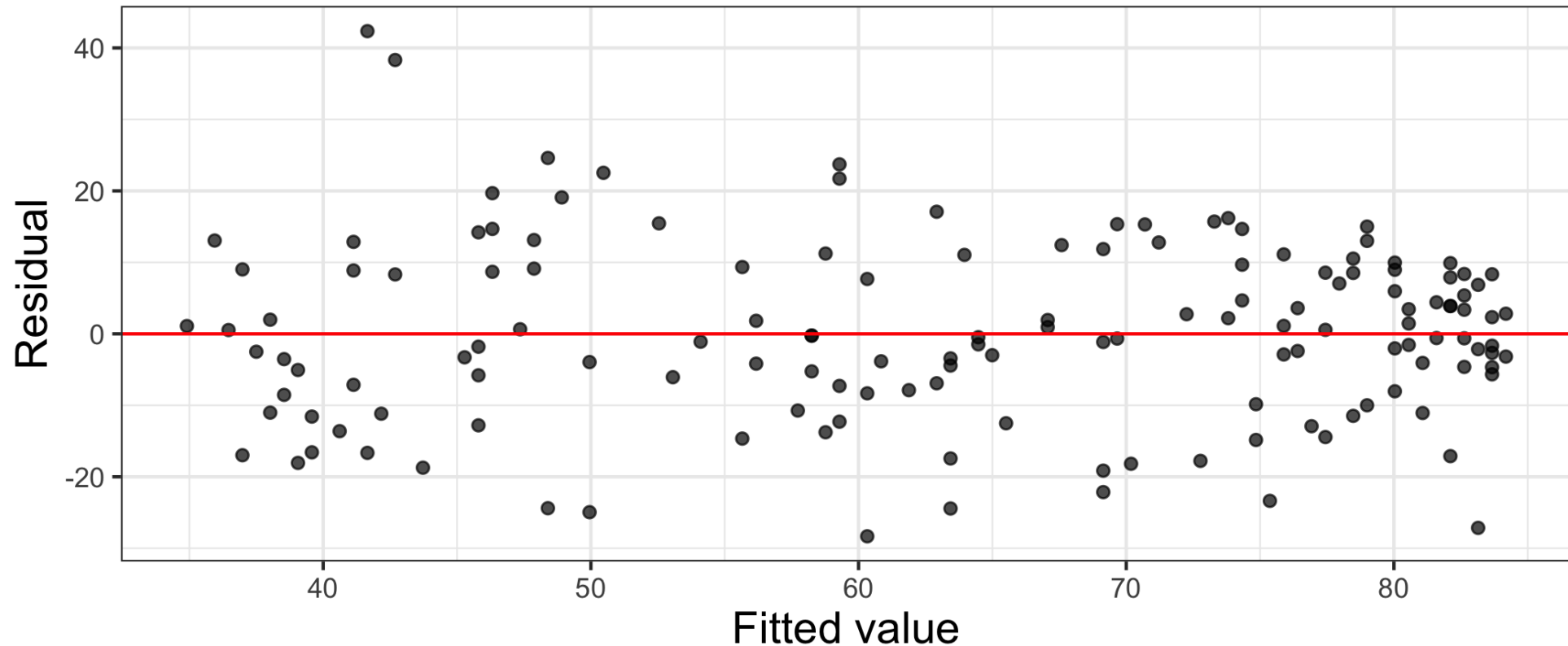


Checking constant variance

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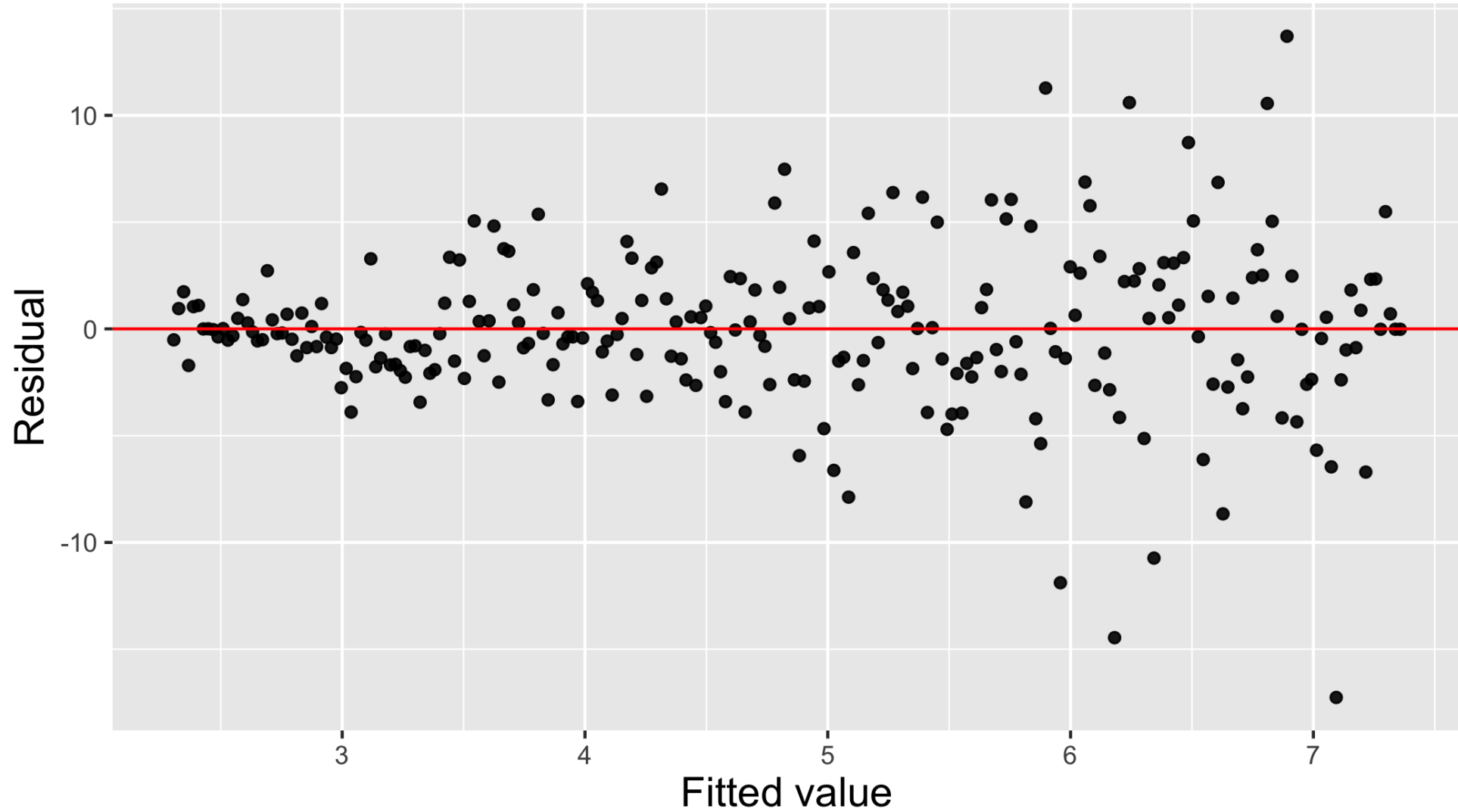


Checking constant variance

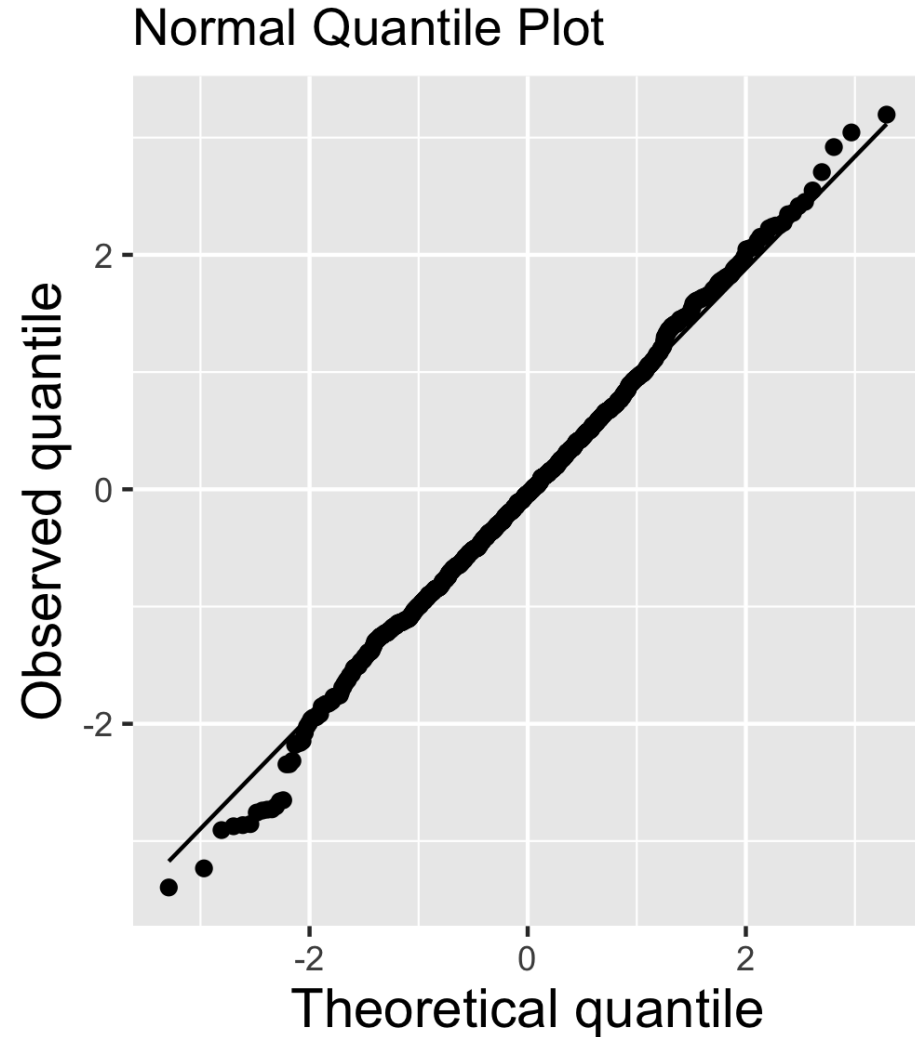
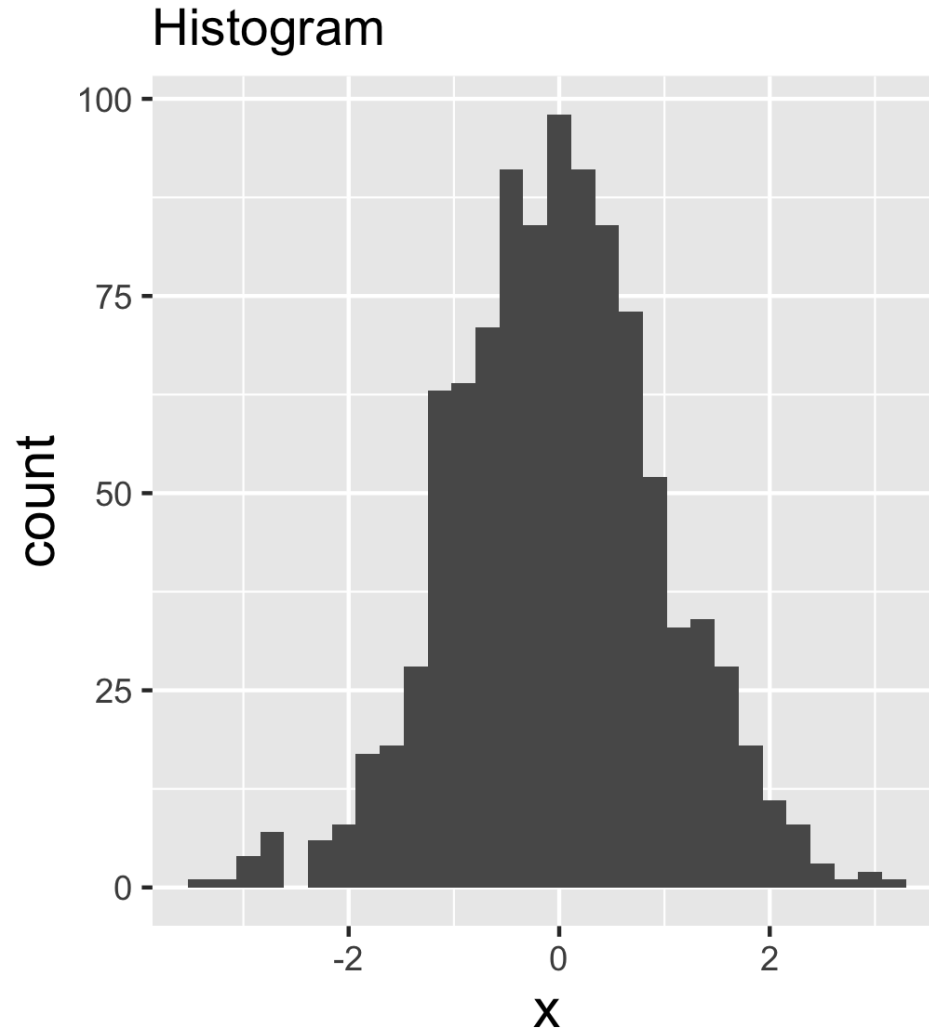


✓ The vertical spread of the residuals is relatively constant across the plot.

✗ Violation: non-constant variance

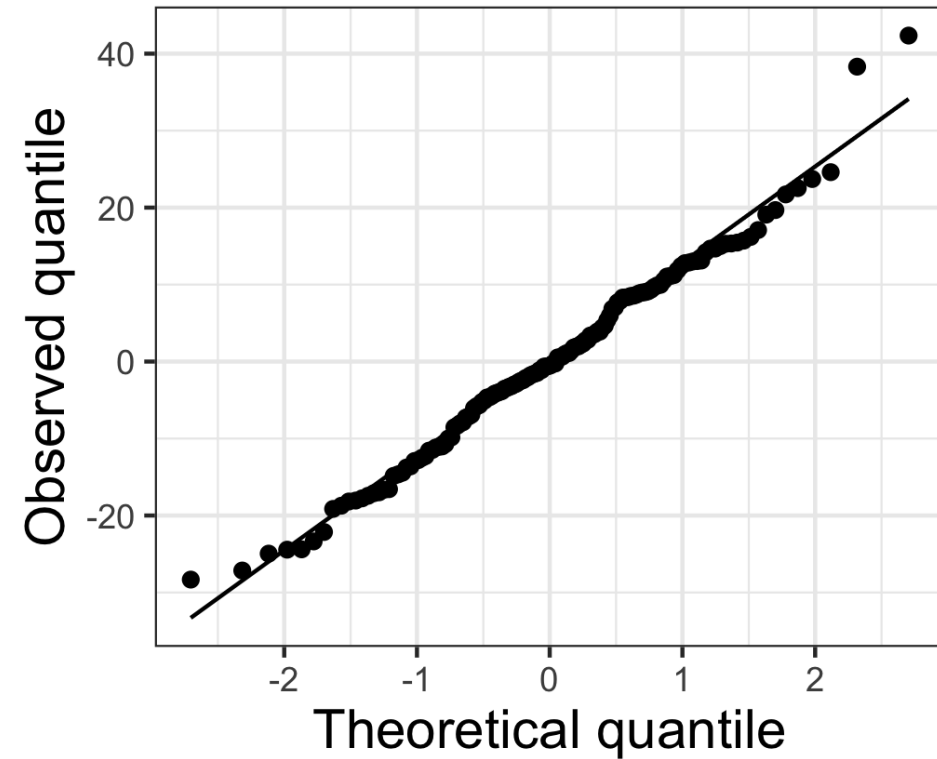
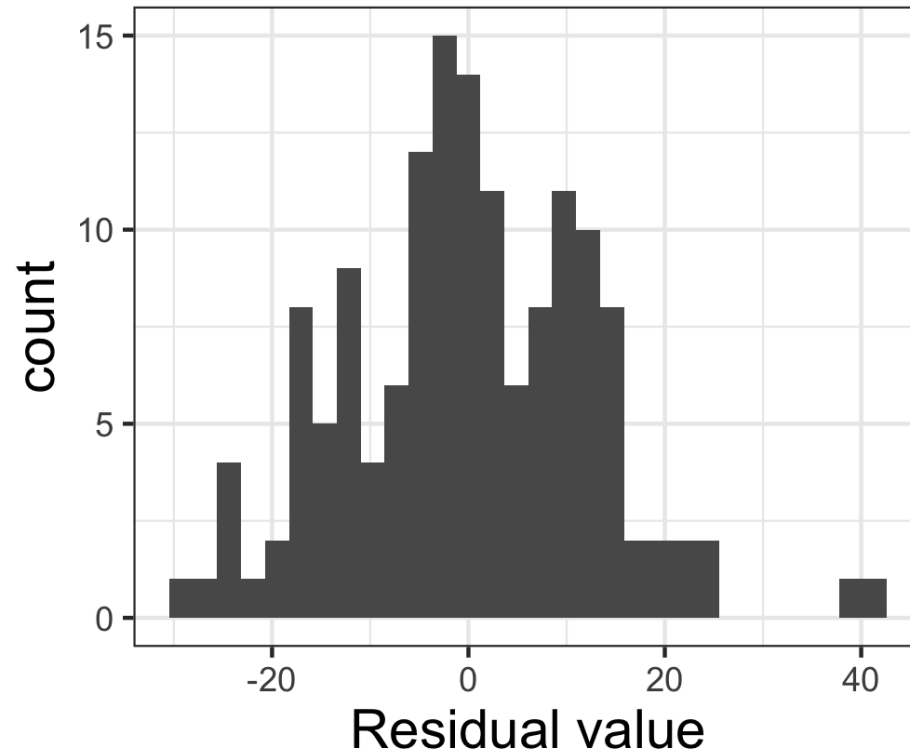


Normal quantile plot

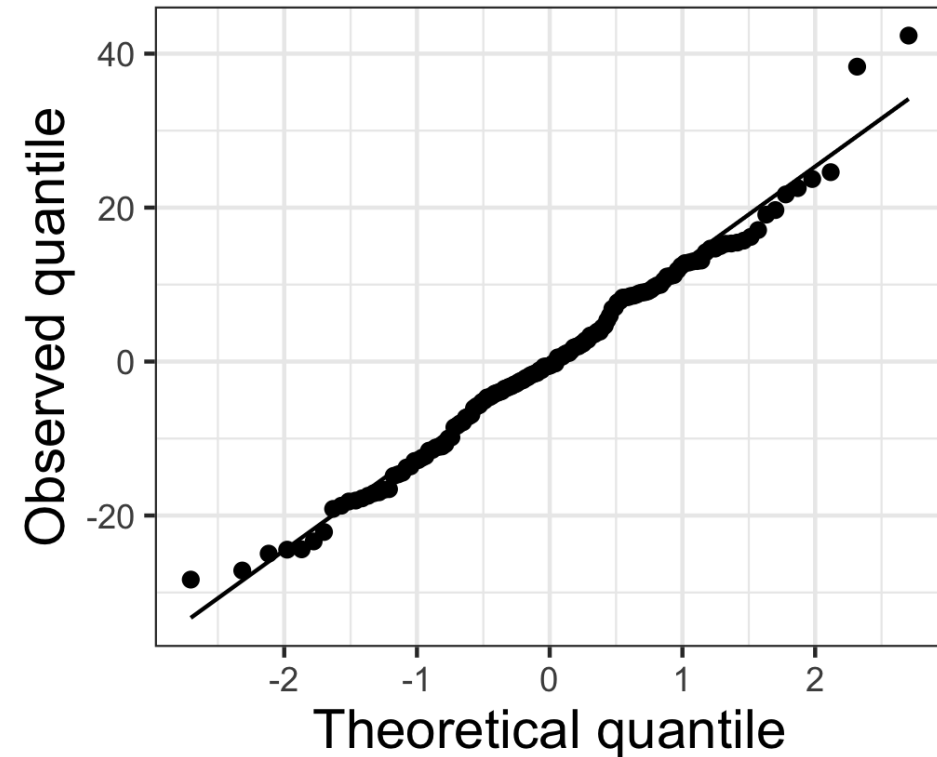
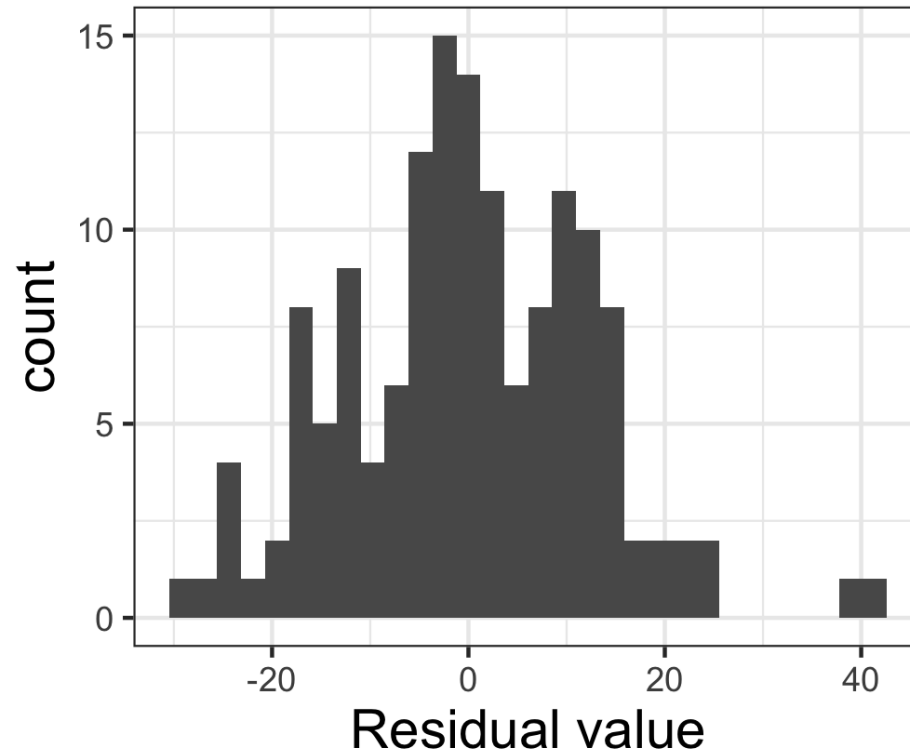


Checking normality

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✅ Points fall along a straight diagonal line on the normal quantile plot.

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- If the data were collected in a particular order, examine a scatterplot of the residuals versus order in which the data were collected.
- ✅ Based on available information, the error for one movie does not tell us anything about the error for another movie.

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- 1 a different model should be proposed.
 - 2 conclusions drawn from the model should be used with caution.
- ✓ If not, the conditions are sufficiently met and we can proceed with the current model.

Recap

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- Used plots of the residuals to check conditions for simple linear regression:
 - **Linearity**
 - **Constant Variance**
 - **Normality**
 - **Independence**