

Simple Linear Regression

Inference

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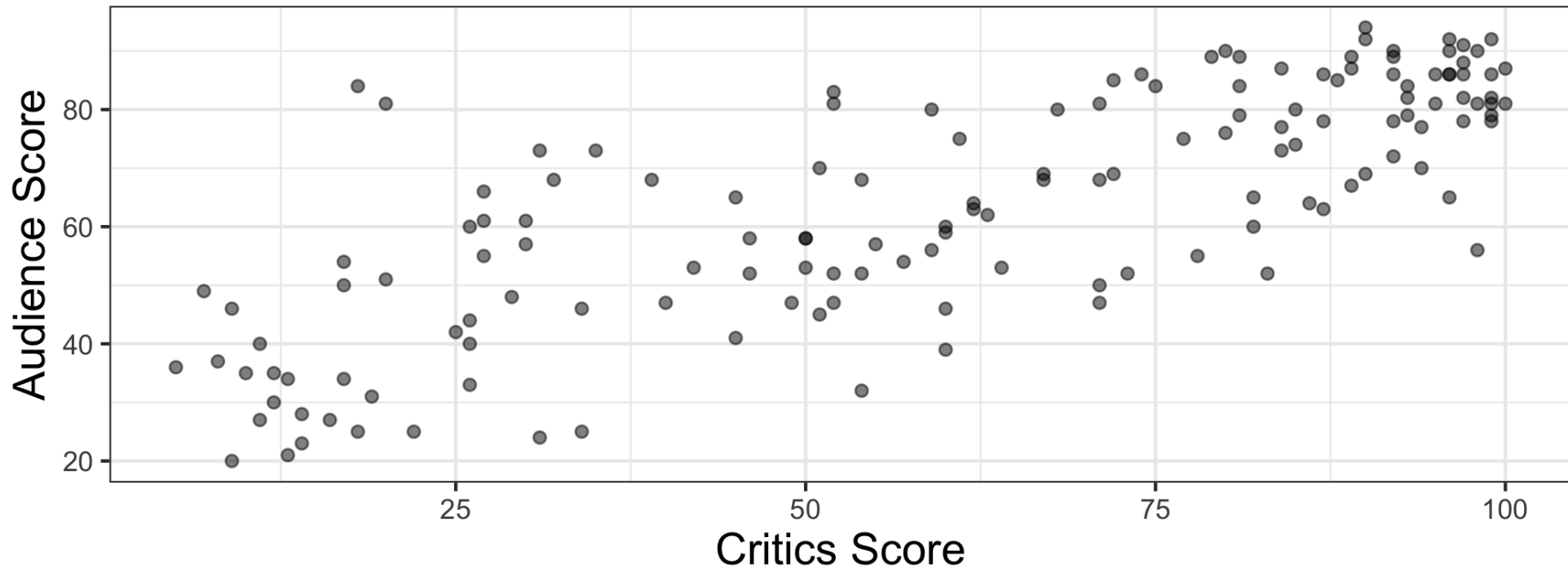
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Topics

- Conduct a hypothesis test for β_1
- Calculate a confidence interval for β_1

Movie ratings data

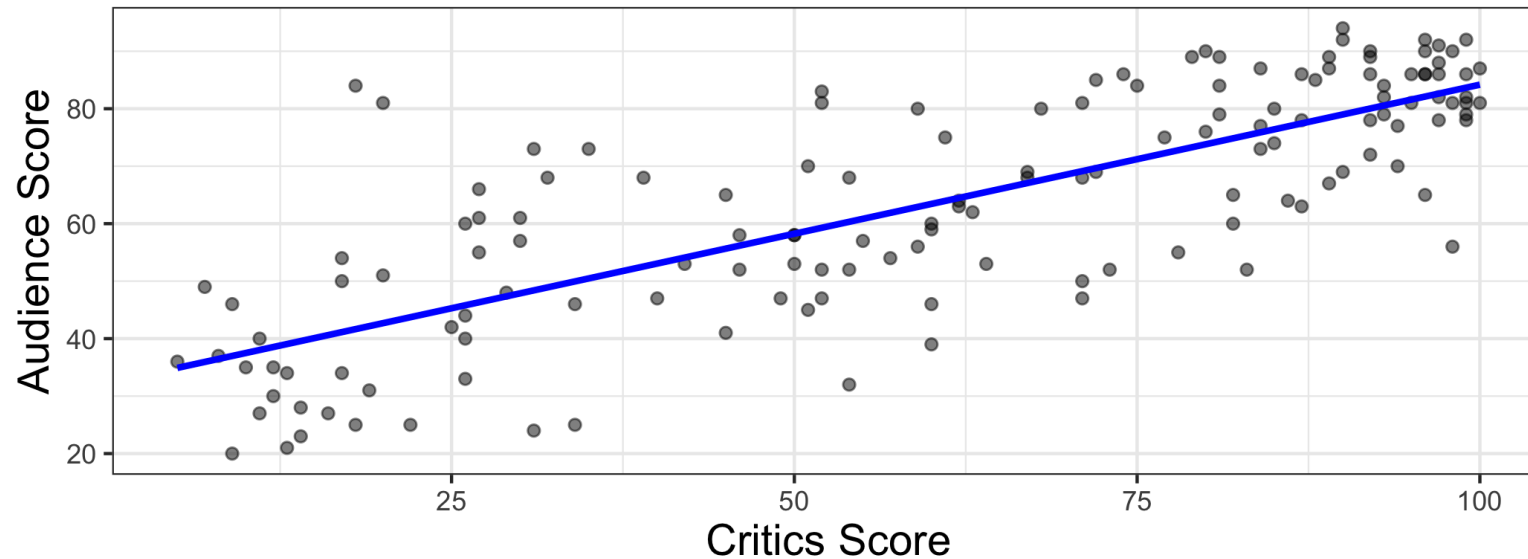
The data set contains the "Tomatometer" score (**critics**) and audience score (**audience**) for 146 movies rated on rottentomatoes.com.



The model

$$\hat{\text{audience}} = 32.316 + 0.519 \times \text{critics}$$

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0



Does the data provide sufficient evidence that β_1 is significantly different from 0?

Outline of a hypothesis test

- 1 State the hypotheses.
- 2 Calculate the test statistic.
- 3 Calculate the p-value.

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- 3 Calculate the p-value.
- 4 State the conclusion.

1 State the hypotheses

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

$$H_0 : \beta_1 = 0$$

$$H_a : \beta_1 \neq 0$$

Null hypothesis

Alternative hypothesis

2 Calculate the test statistic

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

$$\text{test statistic} = \frac{\text{Estimate} - \text{Hypothesized}}{\text{Standard error}}$$

2 Calculate the test statistic

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

$$t = \frac{\hat{\beta}_1 - 0}{SE_{\hat{\beta}_1}}$$

$$\begin{aligned} t &= \frac{0.5187 - 0}{0.0345} \\ &= \mathbf{15.03} \end{aligned}$$

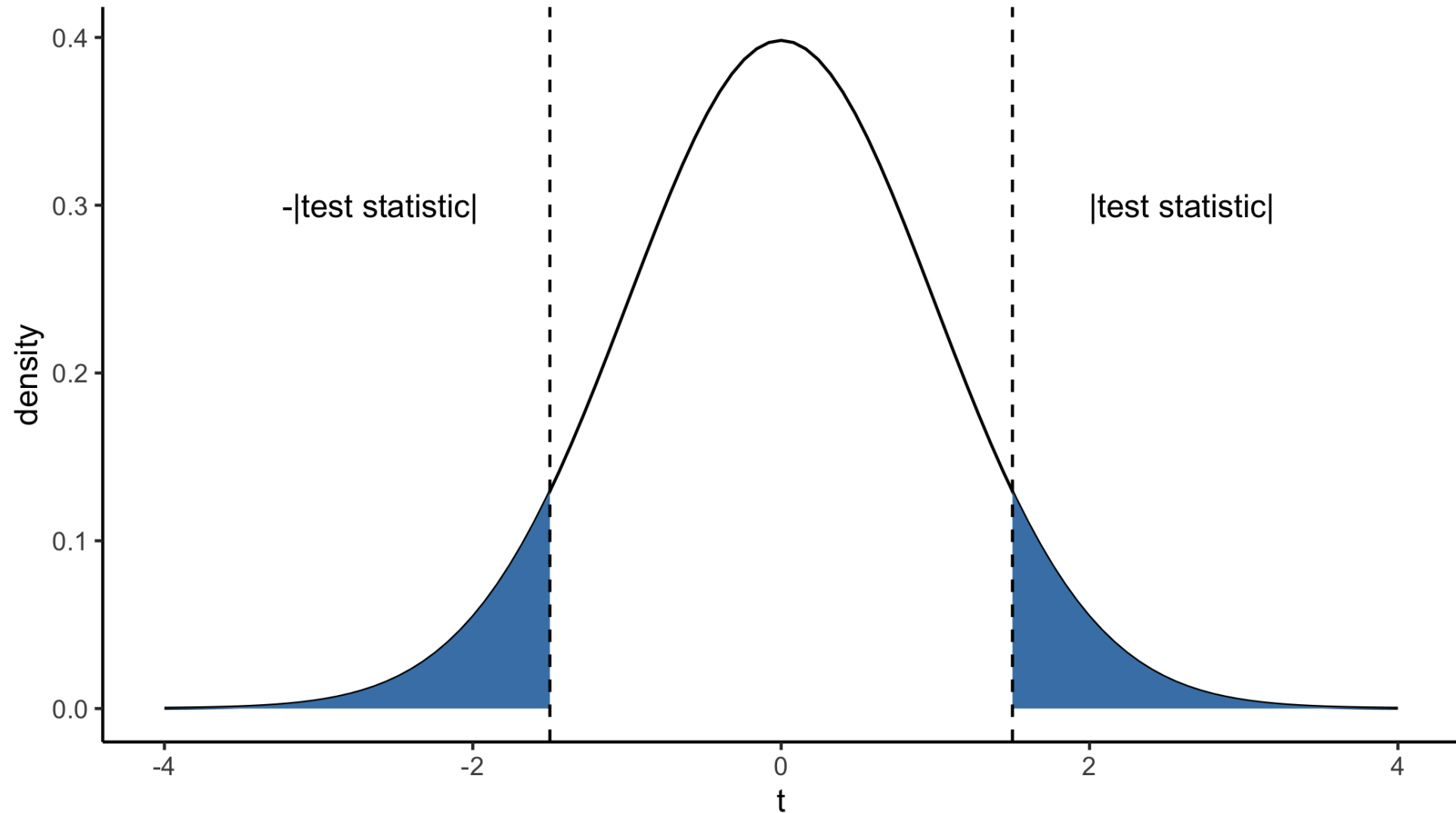
3 Calculate the p-value

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

$$\text{p-value} = P(|t| \geq |\text{test statistic}|)$$

Calculated from a t distribution with $n - 2$ degrees of freedom

3 Calculate the p-value



Understanding the p-value

Magnitude of p-value	Interpretation
p-value < 0.01	strong evidence against H_0
0.01 < p-value < 0.05	moderate evidence against H_0
0.05 < p-value < 0.1	weak evidence against H_0
p-value > 0.1	effectively no evidence against H_0

These are general guidelines. The strength of evidence depends on the context of the problem.

4 State the conclusion

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

The data provide sufficient evidence that the population slope β_1 is different from 0.

There is a linear relationship between the critics score and audience score for movies on rottentomatoes.com.

What is a plausible range of values for the population slope
 β_1 ?

Confidence interval for β_1

$$\text{Estimate} \pm (\text{critical value}) \times \text{SE}$$

$$\hat{\beta}_1 \pm t^* \times SE_{\hat{\beta}_1}$$

t^* is calculated from a t distribution with $n - 2$ degrees of freedom

Calculating the 95% CI for β_1

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

$$\hat{\beta}_1 = 0.519 \quad t^* = 1.977 \quad SE_{\hat{\beta}_1} = 0.035$$

$$0.519 \pm 1.977 \times 0.035$$

$$[0.450, 0.588]$$

Interpretation

$[0.450, 0.588]$

We are 95% confident that for every one percent increase in the critics score, the audience score is predicted to increase between 0.450% and 0.588%.

Recap

- Conducted a hypothesis test for β_1
- Calculated a confidence interval for β_1